Quad and Cavity Position Tolerances

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Introduction

Errors in quadrupole and cavity positions deflect the centroid of the Linac beam, contributing to the aperture requirements in the cavities and quadrupoles. These can be controlled by the use of position electrodes and correction dipoles.

Beam Dynamics

The transverse equation of motion of a particle in the Linac is ($' = \frac{d}{dz}$)

$$x'' + \frac{P'}{P}x' + Kx = 0$$

where $P = mc\beta\gamma$ is the momentum at z and

$$K = K_m + K_{rf}: \quad K_m = \frac{eB'}{Pc}; \qquad K_{rf} = \frac{\pi e T E_0 \cos\phi_s}{\lambda mc^2(\beta\gamma)^3}$$

where ϕ_s is measured from the zero crossing of the electric field. We introduce[1] another variable

$$X = \sqrt{P}x$$

whose equation is

$$X'' + (K + K_{acc})X = 0: \quad K_{acc} = -\frac{[\sqrt{P}]''}{\sqrt{P}} = \left[\frac{eTE_0\sin\phi_s}{mc^2}\right]^2 \frac{\gamma^2 + 2}{4(\beta\gamma)^4}$$

K_{acc} is three orders of magnitude less than K, and will be ignored. The we can use the usual phase amplitude solution for X in terms of locally defined functions α, β, and γ. In terms of the vector

$$\mathbf{Y} = \begin{bmatrix} X \\ \alpha X + \beta X' \end{bmatrix}$$

we can relate the values at two locations z_1 and z_2 by

$$\mathbf{Y}(z_2) = \sqrt{\frac{\beta_2}{\beta_1}}\,\mathbf{R}(\mu_{12})\ \mathbf{Y}(z_1)$$

where $\mathbf{R}(\mu_{12})$ is the rotation matrix for an angle equal to the phase shift between the two points z_1 and z_2

$$\mathbf{R}(\mu_{12}) = \begin{bmatrix} \cos\mu_{12} & \sin\mu_{12} \\ -\sin\mu_{12} & \cos\mu_{12} \end{bmatrix}$$

The change in X' due to a displaced element is

$$\theta = \delta X' = -KL\delta X = -KL\sqrt{P}\delta x$$

so that the change in $\mathbf{Y}$ is given by

$$\delta\mathbf{Y}_m = \Delta_m = \begin{bmatrix} 0 \\ \beta_m\theta_m \end{bmatrix}$$

[1]D. J. Larson, F. E. Mills, and F. T. Cole, IEEE Trans. Nuc. Sci. **NS-32**, 2433, October 1985

The total excitation $\mathbf{Y}_f$ at the end of the linac where $\beta = \beta_f$ is

$$\mathbf{Y}_f = \sum_m \sqrt{\frac{\beta_f}{\beta_m}} \mathbf{R}_{mf} \Delta_m$$

We form the product $\underline{\mathbf{Y}}_f \mathbf{Y}_f$,where $\underline{\mathbf{Y}}_f$ is the transpose of $\mathbf{Y}_f$

$$\underline{\mathbf{Y}}_f \mathbf{Y}_f = X^2 + (\alpha_f X + \beta_f X')^2 = \sum_{m;n} \frac{\beta_f}{\sqrt{\beta_m \beta_n}} \Delta_m \underline{\mathbf{R}}_{mf} \mathbf{R}_{nf} \Delta_n$$

$$= \sum_{m;n} \beta_f \sqrt{\beta_m \beta_n} \theta_m \theta_n \cos\mu_{mn}$$

Now we average $\underline{\mathbf{Y}}_f \mathbf{Y}_f$ over the ensemble of errors. $\langle \underline{\mathbf{Y}}_f \mathbf{Y}_f \rangle$ is the expectation value of the mean square of X, or for x,

$$\sigma_x^2 = \frac{\langle \underline{\mathbf{Y}}_f \mathbf{Y}_f \rangle}{P_f}$$

Uncorrelated errors

If the position errors are uncorrelated, then each error contributes separately, and

$$\sigma_x^2 = \beta_f \sum_m \beta_m \frac{P_m}{P_f} (KL)_m^2 (\delta x_m)^2$$

In the quad lattices presently under consideration[1], all 28 quads are the same length, and there are only four slightly different strengths, all 8 cm length and about 20 T/m gradient. This provides a value of β which almost increases as P (actually closer to β) so that the beam is slightly damped in size during acceleration. Then if $Q_m = (KL)_m \frac{P_m}{P_f} = \frac{eB'L}{P_f c}$, Q_{quad} = 20T/m•0.08m/3.2Tm) = 0.5 m^{-1} for the quads, and we define Q_{rf} in analogy to Q_{quad}

$$\sigma_x^2 = \beta_f \sum_m \frac{P_f \beta_m}{P_m} Q_m^2 (\delta x_m)^2$$

The following table will help clarify this sum. There are 7 modules, each with four $15\beta\lambda/2$ cavities, two F and two D quads.

[1]J. MacLachlan, private communication

Energy	116MeV	400MeV
P	480MeV/c	954MeV/c
β_{max}	8.35 m	11.72 m
β_{min}	1.73 m	3.1 m
$\beta_{max}P_f/P$	16.7 m	11.72 m
$\beta_{min}P_f/P$	3.5 m	3.1 m
Q_{quad}	0.5 m^{-1}	0.5 m^{-1}
$\beta\lambda/2$	0.0851 m	0.1328 m
K_{rf}	0.199 m^{-2}	0.026 m^{-2}
$15\beta\lambda/2K_{rf}$	0.24 m^{-1}	0.052m^{-1}
Q_{rf}	0.12 m^{-1}	0.052m^{-1}

It is seen that the cavity position errors are almost insignificant compared to the quad errors, most of their contribution coming from those at low energy. The result of completing the sum relates the rms of the expected beam position error at the F quad at 400 MeV to the rms location errors, and can be expressed as an "amplification factor", which is about 27. That is,

$$\sigma_x = 27\ \delta x_{rms}$$

This is large enough that steering corrections will be needed. In actual fact, the 4 cavity, four quad modules will undoubtably be mounted on "girders", or "strongbacks", in which case the random location error can be reduced to <0.1 mm., say 0.05 mm. Then the random errors will contribute about 1.3 mm to the rms position error. Now, however, we must concern ourselves with the girder position errors, which are probably larger, say 0.25 mm.

Correlated Errors

If we mount each module of four cavities and four quadrupoles on a girder, then we ascribe to each girder a position error. (More realistically there is an error which is distributed over the girder, but we will ignore that for now and assume the same error for all elements on the girder.) Then the double sum above becomes a sum over elements on each girder and a sum over all girders. The result, for quads only and ignoring momentum changes on a girder, is

$$\sigma_x^2 = \beta_f \sum_{girders} 2Q^2(\delta x_g)^2 \frac{P_f}{P_g}\left[(\beta_{max}+\beta_{min})(1+\cos\sigma) - \sqrt{\beta_{max}\beta_{min}}(3\cos\sigma/2+\cos 3\sigma/2)\right]$$

Here σ is the phase advance per FODO period. We will take σ to be 60°, but the result is not very sensitive to that choice. The result is again an amplification factor which is about 18 for the parameters in the table above. That is,

$$\sigma_x = 18\ \delta x_g$$

Correction elements

In order to keep the beam in the aperture and avoid beam loss, we will need to measure beam positions and install correction dipoles. If we put one such device for each plane per girder, then the correction magnet needs to handle several times the implied angle for each girder determined from the σ_x above. Suppose we achieve the tolerances suggested above, that is 0.05 mm for components on the girder and 0.25 mm for the girders themselves. Then σ_x is about 5 mm, and the rms of the expected angular error is (β = 12 m) 0.4 mrad. Each girder then contributes about 0.15 mrad. Remembering that the errors are damped as $P^{1/2}$, we see that 1 mrad correctors for each plane should be adequate.

The placement of the position electrodes and correctors is as important as their strength. The electrode and corrector should be located in the same place, and at an F quad (at β_{max}) for the plane in question. It follows that at least one of these will be located at a bridge coupler. It appears that there is enough space in the bridge coupler to do this, if the correction magnet is mounted around the position electrode.